

Fractal Bubble Cosmology: A concordant cosmological model? *

Juliana Kwan¹, Matthew J. Francis^{1,2} & Geraint F. Lewis¹

¹*Sydney Institute for Astronomy, School of Physics, A28, University of Sydney, NSW 2006, Australia*

²*SISSA, International School of Advanced Studies, via Beirut 2-4, 34014, Trieste, Italy*

26 November 2009

ABSTRACT

The Fractal Bubble model has been proposed as a viable cosmology that does not require dark energy to account for cosmic acceleration, but rather attributes its observational signature to the formation of structure. In this paper it is demonstrated that, in contrast to previous findings, this model is not a good fit to cosmological supernovae data; there is significant tension in the best fit parameters obtained from different samples, whereas Λ CDM is able to fit all datasets consistently. Furthermore, the concordance between galaxy clustering scales and data from the cosmic microwave background is not achieved with the most recent supernova compilations. The validity of the FB formalism as a sound cosmological model is further challenged as it is shown that previous studies of this model achieve concordance by requiring a value for the present day Hubble constant that is derived from supernovae data containing an arbitrary distance normalisation.

Key words: cosmological parameters — cosmology: observations — cosmology: theory

1 INTRODUCTION

The discovery of cosmic acceleration, (Riess et al. 1998; Perlmutter et al. 1999), and its subsequent explanation as dark energy has resulted in the proposal of many alternate models to the current cosmological paradigm Λ Cold Dark Matter (Λ CDM) as the nature of dark energy remains elusive. There are three main alternatives to dark energy: either General Relativity is not the correct description of gravity on large scales, introduce a higher dimensional theory, or the metric that describes the universe is not required to be maximally symmetric (see Durrer & Maartens 2008, for a review). The Fractal Bubble (FB) model (Wiltshire 2007a,b) falls into the last category, as dark energy is replaced by ‘quasi-local gravitational energy’ arising from changes in the curvature of spacetime due to an inhomogeneous distribution of matter. In Leith et al. (2008), it is claimed that the FB model achieves a concordant set of cosmological parameters using supernova data from Riess et al. (2007), the angular scale of the sound horizon from WMAP1 (Bennett et al. 2003) and the comoving spatial separation of the correlation function in SDSS (Eisenstein et al. 2005).

Although the vast majority of inhomogeneous cosmological models have been shown to be inconsistent with observations or rely on a particular choice of coordinates (Ishibashi & Wald 2006; Ziblin et al. 2008; Caldwell & Stebbins 2008), the FB model remains unchallenged. In this paper, we examine the ability of the FB model to produce a concordant set of cosmological parameters. In particular, we address the claim of Leith et al. (2008) that their two parameter fits for the distance modulus given by the FB model using the supernova type Ia (SNe Ia) sample of Riess et al. (2007) supports a concordance of observational evidence for the FB model. Additionally, we demonstrate a number of discrepancies including the inability of the FB model to consistently describe both the Riess et al. (2007) sample and the Union supernova compilation (Kowalski et al. 2008) or the Constitution set (Hicken et al. 2009) despite none of these SNe Ia samples being in tension. Leith et al. (2008) also omit to calibrate the Riess et al. (2007) SNe Ia data to a distance scale when fitting for the Hubble constant, H_0 .

2 BACKGROUND

The FB model (Wiltshire 2007a,b) is a two scale model with a local régime for over or underdense regions and another

* Research undertaken as part of the Commonwealth Cosmology Initiative (CCI: www.thecci.org), an international collaboration supported by the Australian Research Council

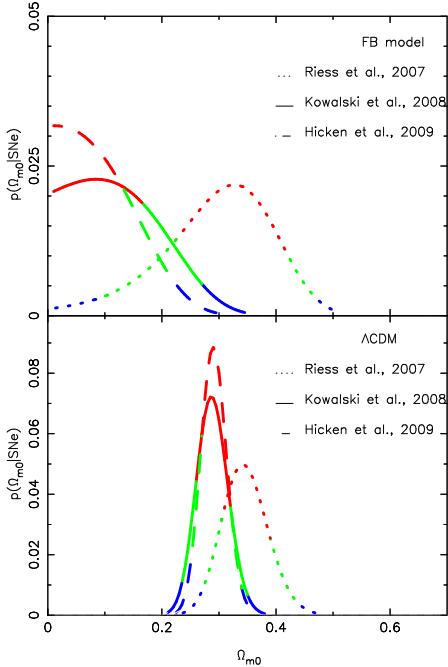


Figure 1. Top: Marginalised posterior distributions for the FB model using the Union compilation (Kowalski et al. 2008) in solid lines, the Gold SNe Ia sample from Riess et al. (2007) in dotted lines, and the Constitution set (Hicken et al. 2009) in dashed lines. The colours correspond to the confidence limits, red is 1σ , green is 2σ and blue is 3σ . Bottom: As above except using ΛCDM .

for the volume average, which utilises the Buchert averaging scheme (Buchert 2000). We use an overbar to indicate volume averaged quantities as opposed to ‘dressed’ parameters measured by observers in galaxies, termed ‘walls’ in the FB model. The volume averaged scale factor, \bar{a} , and the void fraction, f_v , defined as the total volume in void regions, evolve according to the Buchert equations as follows:

$$\frac{\dot{\bar{a}}^2}{\bar{a}^2} + \frac{\dot{f}_v^2}{9f_v(1-f_v)} - \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2} = \frac{8\pi G}{3}\bar{\rho}_0\frac{\bar{a}_0^3}{\bar{a}^3}, \quad (1)$$

$$\ddot{f}_v + \frac{\dot{f}_v^2(2f_v-1)}{2f_v(1-f_v)} + 3\frac{\dot{\bar{a}}}{\bar{a}}\dot{f}_v - \frac{3\alpha^2 f_v^{1/3}(1-f_v)}{2\bar{a}^a} = 0, \quad (2)$$

where α is the normalisation constant of the curvature energy. Equation 1 is equivalent to writing: $\bar{H}^2 - \bar{\Omega}_q \bar{H}^2 - \bar{\Omega}_k \bar{H}^2 = \bar{\Omega}_m \bar{H}^2$, since the volume averaged Hubble constant is defined as $\bar{H} = \dot{\bar{a}}/\bar{a}$ and the normalised volume averaged energy densities are given by:

$$\bar{\Omega}_q = -\frac{\dot{f}_v^2}{9f_v(1-f_v)\bar{H}^2} \quad (\text{backreaction}), \quad (3)$$

$$\bar{\Omega}_k = \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2 \bar{H}^2} \quad (\text{curvature}), \quad (4)$$

$$\bar{\Omega}_m = \frac{8\pi G}{3}\bar{\rho}_0\frac{\bar{a}_0^3}{\bar{a}^3 \bar{H}^2} \quad (\text{matter}). \quad (5)$$

Information about the distribution of matter is encoded in the void fraction, f_v , which is directly proportional to the dressed normalised matter density, $\Omega_{m,0}$, as $\Omega_{m,0} \approx \frac{1}{2}(1-f_{v,0})(2+f_{v,0})$ (Wiltshire 2007b). Although a back-reaction term occurs in the above equations, this is not the principal mechanism that creates cosmic acceleration, but

rather this is achieved through ‘quasi-local’ gravitational energy quantified by the curvature energy term. Such gravitational energy can not be localised in the stress energy tensor and in the FB model gives rise to an apparent cosmic acceleration as measurements are distorted when signals pass through regions of varying curvature and it cannot be assumed that the clocks of all observers are synchronous independent of location. In the absence of homogeneity, volume averaged quantities must be transformed back to what observers in overdensities like ourselves would measure. The FB model, chooses to foliate spacetime such that there is a lapse function, defined as $\bar{\gamma}(\tau) \equiv dt/d\tau$, where τ is the proper time of an observer in a wall or a void and t is the volume averaged or cosmic time, which quantifies the difference between clocks of two different observers depending on their location. If the lapse function in walls is significant, then the FB model predicts that a spurious cosmic acceleration will be detected by failing to account for an inhomogeneous matter distribution through the assumption of a Friedmann-Lemaître-Robertson-Walker (FLRW) background and a recalibration of observations for the difference in clocks between walls and the volume average is necessary. This difference may be as large as $\bar{\gamma} \approx 1.38$ to achieve the level of concordance seen in Leith et al. (2008).

Unlike previous approaches with inhomogeneous cosmologies, much of the physics behind the FB model is essentially alien to the standard model of modern cosmology. An extension to the equivalence principle (Wiltshire 2008) is required to explain the anomalously large value of $\bar{\gamma}$ necessary to produce the best fit parameters quoted in Leith et al. (2008) and Wiltshire (2007a). Furthermore, neither the Newtonian limit or Birkhoff’s theorem is relevant in this cosmology. Leaving aside these conceptual issues, we have instead focused on establishing the compatibility of currently available observational data with the FB model.

3 TENSION WITH SUPERNOVA DATA

3.1 Constraints from new data sets

The Union supernova compilation (Kowalski et al. 2008) is primarily composed of SNe Ia catalogs from literature that have been reanalysed in an uniform manner to reduce systematics, with some further cuts imposed to exclude data of insufficient quality. The compilation contains a total of 307 SNe Ia, of which 27 are drawn from Riess et al. (2007) after the final cut and a further eight are derived from new low-redshift observations. Its constituent catalogues have also been shown to be consistent with one another under this new analysis [see Figure 9 of Kowalski et al. (2008)]. More recently, the entire Union compilation has been amalgamated with an additional sample of 90 new CfA3 SNe Ia into the Constitution set, increasing the amount of low redshift data by a factor of 2.6 (Hicken et al. 2009). Like the Union compilations, the Constitution set is uniformly reduced through the use of a single light curve fitter for all SNe Ia. Clearly, both the Union compilation and the Gold sample of Riess et al. (2007) are entirely consistent with the Constitution set.

We test the ability of the FB model to constrain cosmology with the new Union and Constitution SNe Ia data

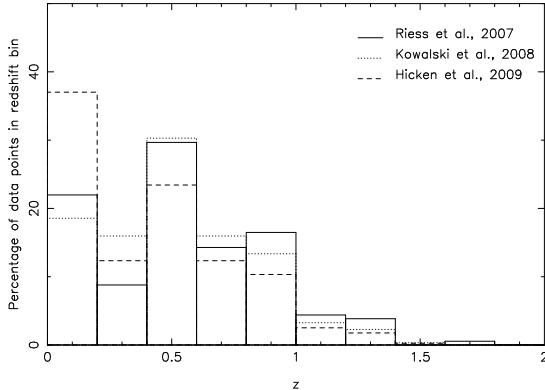


Figure 2. Percentage of SNe Ia from Gold (solid), Union (dotted) and Constitution (dashed) samples falling into redshift bins of width $\Delta z = 0.2$. There are 397 SNe Ia in the Constitution set, 307 SNe Ia in the Union compilation and 182 SNe Ia in the Gold data set of Riess et al. (2007).

sets, as well as the Gold sample from Riess et al. (2007). This last sample was analysed by Leith et al. (2008), but this study suffered from a number of problems as, detailed in Section 3.2, which we rectify in this work. The best fit cosmological parameters are found from calculating the reduced chi squared, χ^2 , based on the distance modulus given by $\mu = 5 \log_{10}(d_L) + 25$ for Λ CDM and FB model to all three samples of SNe Ia. The luminosity distance d_L , is defined as:

$$d_L = \begin{cases} (1+z)a_0 \int_t^{t_0} \frac{dt}{a(t)} & (\Lambda\text{CDM}), \\ \frac{\bar{\gamma}}{\bar{\gamma}_0} \bar{a}_0 (1+z) (1-f_v)^{1/3} \int_t^{t_0} \frac{dt}{\bar{\gamma}(1-f_v)^{1/3} \bar{a}} & (\text{FB}). \end{cases}$$

There are two free parameters of interest in the FB model that are used to calculate the distance modulus; the cosmology is fully constrained after specifying the initial void fraction f_{vi} , the dressed Hubble constant today, H_0 , and the initial ratio of the expansion rates as measured in a wall and a void, h_{ri} ; the latter remains fixed for all fits. We have used the same priors as Leith et al. (2008) on f_{vi} and h_{ri} , namely $10^{-5} < f_{vi} < 10^{-2}$ (and hence $0.01 \leq \Omega_{m,0} < 0.95$) and $h_{ri} = 0.99999$ at last scattering, with $50 \leq H_0 \leq 75$ $\text{km s}^{-1} \text{Mpc}^{-1}$. The Λ CDM priors are the same on H_0 and cover a similar range on $\Omega_{m,0}$: $0 \leq \Omega_{m,0} \leq 1$. The value of H_0 is treated as a nuisance parameter, which we marginalise over, since it cannot be generally assumed that the absolute magnitudes of the SNe Ia are known. This results in a single parameter fit for $\Omega_{m,0}$, which is directly related to f_v .

The χ^2 statistic is converted to a likelihood, \mathcal{L} , via the relationship, $\mathcal{L} \propto \exp(-\chi^2/2)$. Bayesian inference then gives the resultant posterior distributions as presented in Figure 1 for both cosmological models, with the best fit values shown in Table 1. For Λ CDM, all three distributions give a consistent set of cosmological parameters that are in agreement, with the 1σ confidence regions significantly overlapping. This does not occur for the FB model; the 1σ limits derived for each SNe Ia sample do not all coincide. We must consider the 2σ confidence region before we can find a value of $\Omega_{m,0}$ that agrees with between the Union compilation or the Constitution set and the Gold sample of Riess et al. (2007). In fact, the best fit value for the Gold sample from Riess et al. (2007) is ruled out at 3σ when performing the same analysis for the Union compilation and is beyond the 3σ limit for the Constitution set. (See Table 1

for the complete set of best fit parameters and 1σ errors). However, there is good agreement between the best fit FB parameters for Constitution set and the Union compilation, which is reassuring; the reason for this discordance lies with the model rather than an anomaly in the SNe Ia sample. If this model is to be acknowledged as a viable alternative to Λ CDM, then it seems that we must also accept that there is no consistent observational evidence from SNe Ia for the FB model or that these two SNe Ia samples are in tension with the Riess et al. (2007) data set. However, Kowalski et al. (2008) found a high degree of consistency between the samples used in the construction of the Union compilation, albeit with mild tension between the Krisciunas et al. (2004a,b, 2001) and Hamuy et al. (1996) samples and the other SNe Ia data when comparing $d\mu/dz$. Crucially, the more recent and populous samples break the concordance of cosmological tests for the FB model; clearly the best fit values for $\Omega_{m,0}$ as shown in Table 1 do not overlap with those obtained from galaxy clustering statistics and the angular scale of the sound horizon found by Leith et al. (2008), which require $0.27 \lesssim \Omega_{m,0} \lesssim 0.37$. Furthermore, although the Bayes factor (Trotta 2008) between the FB model and Λ CDM with the Riess et al. (2007) Gold data set marginally favours the former [$\ln B = 0.27$, Leith et al. (2008)], the FB model is weakly disfavoured when using the Union compilation or the Constitution set ($\ln B = -1.38, -1.469$, respectively) under the same classification scheme on the strength of evidence. As the treatment of systematics improve and sample sizes grow, Λ CDM is increasing being favoured by the data while the best fit parameters are moving away from those required by the FB model for concordance. The deterioration of the evidence towards the FB model in terms of the Bayes factor may be attributed to the width of the posterior in $\Omega_{m,0}$, which remains significantly larger than that of the corresponding data set under Λ CDM.

3.2 Distance normalisation?

Figure 2 of Leith et al. (2008) depicts a concordance diagram, represented in the $\Omega_{m,0}$ - H_0 plane, for the FB model, with confidence limits from SNe Ia using the Gold data of Riess et al. (2007), the proper distance to the sound horizon seen in SDSS, and the angular scale of the sound horizon at decoupling. However, a number of issues regarding the analysis of these observational constraints remain unaddressed by the authors, which, when corrected, falsify the concordance. The most serious of these is that the SNe Ia data used to produce this diagram contains an arbitrary value of H_0 without the appropriate correction to the distance moduli. Thus, it is misleading to present confidence limits in Figure 2 of Leith et al. (2008) that depend on H_0 , since these do not represent its true value. Instead, the concordance diagram in Leith et al. (2008) should be shown with either H_0 as a nuisance parameter, which is marginalised or with the recommended calibration of Riess et al. (2007).

For the Gold sample, Riess et al. (2007) suggest using a systematic subtraction of 0.32 mag^1 to use the same

¹ Magnitude calibration and data can be obtained online at braeburn.pha.jhu.edu/~ariess/R06/sn_sample

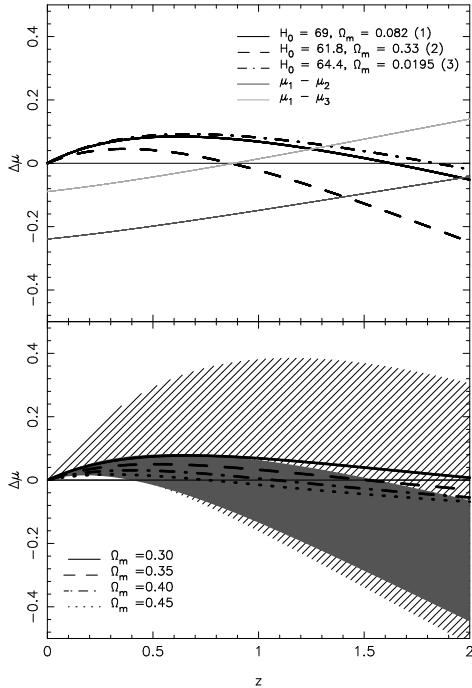


Figure 3. *Left:* Residuals from an empty FLRW universe with same value of H_0 as the FB model with best fit parameters from the Union compilation (solid), Constitution set (dot-dashed) and Gold Riess et al. (2007) sample (dashed). The difference in the distance moduli between the best fit values found by Leith et al. (2008) and those from the more recent compilations are shown in light grey (Union) and dark grey (Constitution) *Right:* Range of residuals from an empty FLRW universe with same value of H_0 for FB model (solid grey) and Λ CDM (hatched black) for $0.1 < \Omega_{m,0} < 0.5$. Any residual from an empty universe using either the FB model or Λ CDM, with the above parameters, will fall into the corresponding shaded area. The curves show the difference between the distance moduli of the FB model and Λ CDM for the same value of H_0 for $\Omega_{m,0} = 0.30$ (*solid*), 0.35 (*dashed*), 0.40 (*dot-dashed*), 0.45 (*dotted*). There is a preferred range of parameters, namely $0.35 \lesssim \Omega_{m,0} \lesssim 0.4$ for which the difference between the two models is minimised.

Cepheid scale as Riess et al. (2005) if the value of H_0 is of interest and this is unnecessary for fits to dynamic quantities only. An arbitrarily chosen value of H_0 is inserted into the data, since the absolute magnitude of a SNe Ia is not known until an appropriate distance scale, such as that obtained from Cepheid luminosities, is applied. This correction to the distance moduli has not been taken into account by Leith et al. (2008), who claim to follow the Cepheid calibration of Sandage et al. (2006) but no corrections have been applied at all. Indeed, when performing our fits to the Riess et al. (2007) Gold sample, we are able to reproduce the same best fit value of H_0 as quoted in Leith et al. (2008) (within 0.1 error) without accounting for the arbitrary distance normalisation at all. The authors, however, have interpreted the value of H_0 in the Riess et al. (2007) sample as a physical parameter that constrains cosmology, rather than as a value chosen by Riess et al. (2007). Thus the fit to H_0 in Figure 2 of Leith et al. (2008) is meaningless without calibrating the data. It is plausible that the arbitrary normalisation chosen by Riess et al. (2007) happens to

coincide with the Sandage distance scale, but no justification is given for the choice of calibration in Leith et al. (2008). Interestingly, using the Cepheid calibration recommended by Riess et al. (2007) gives a best fit value of $H_0 = 71.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $\Omega_{m,0}$ unchanged, since changing H_0 only shifts the scale of the Hubble diagram. This no longer produces a concordant set of cosmological parameters; in fact it is stated in Leith et al. (2008) that for any value of H_0 greater than $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, the observational constraints from the proper distance to the sound horizon and its angular scale at decoupling would no longer agree.

4 A COINCIDENCE PROBLEM FOR THE FB MODEL

To understand why the cosmology of the FB model changes dramatically for the Union compilation and the Constitution set while Λ CDM remains consistent, we have analysed the redshift distributions of these two SNe Ia catalogues. Figure 2 shows that the Gold sample of Riess et al. (2007) contains a slightly higher percentage of high redshift SNe Ia than in the other samples but there are significantly more low redshift samples in the latter, particularly in the $0.0 \leq z < 0.2$ range for the Constitution set and $0.35 \lesssim z < 0.4$ range for the Union compilation. Furthermore, it should be unsurprising that the FB model is able to fit both the Constitution set and the Union compilation within 1σ error; Figure 2 also shows that the redshift distribution of these two samples are very similar, especially in the high- z régime where the Constitution set is dominated by data taken from the Union compilation.

The reason behind the shift in parameter space is made apparent by considering the range over which the newer compilations contain more data. This is coincident with the region in which the three different best fits for the FB model diverge the most (Figure 3; left). The residuals have been taken relative to an empty universe with $\Omega_{m,0} = 0$, $\Omega_{\Lambda,0} = 0$, $\Omega_{k,0} = 1$, and the same H_0 to remove the uncertainty in its value. For the Union and Constitution sets, the FB model requires a much greater amount of acceleration to fit the data and indeed the period of apparent acceleration occurs at $z \approx 1.65$ and $z \approx 1.85$ respectively. As the cosmological constraints become progressively tighter in the more recent SNe Ia samples, the FB model is increasingly pushed to higher values of curvature energy. Indeed, the best fit parameters from the Union and Constitution sets imply that the universe is almost entirely composed of voids with an almost negligible amount of collapsed structure. As remarked in the previous section, the FB model has a flatter, wider posterior than Λ CDM, which implies that a greater range of the parameter space can fit the data. This can be attributed to the limited range of behaviour that the FB model can exhibit in terms of changes to the distance modulus when f_{vi} is altered. This is demonstrated in the difference between the range that is spanned by the residuals for the FB model (solid grey) and Λ CDM (hatched black) for $0.1 < \Omega_{m,0} < 0.5$ (Figure 3, right). Although the prior on f_{vi} spans ≈ 3 orders of magnitude, the residuals for the FB model have a substantially truncated régime of acceleration in comparison to Λ CDM. This partly accounts for the discordance seen in the confidence limits for this model

Table 1. Best fit parameters for Λ CDM and Fractal Bubble model to SNe Ia data. Distance normalisation is arbitrary for the Union compilation and the Constitution set. Best fit values of $\Omega_{m,0}$ and H_0 are subject to an additional error of $\pm 5 \times 10^{-3}$ and ± 0.1 respectively due to grid spacing.

Model	SNe Ia sample	$\Omega_{m,0}$	H_0 (km/s/Mpc)
FB	Riess et al. (2007) ¹	$0.33^{+0.07}_{-0.11}$	61.8
Λ CDM	Riess et al. (2007) ¹	$0.34^{+0.045}_{-0.035}$	62.8
FB	Riess et al. (2007) ²	$0.33^{+0.07}_{-0.11}$	71.6
Λ CDM	Riess et al. (2007) ²	$0.34^{+0.045}_{-0.035}$	72.8
FB	Kowalski et al. (2008)	$0.08^{+0.08}_{-0.08}$	—
Λ CDM	Kowalski et al. (2008)	$0.29^{+0.03}_{-0.025}$	—
FB	Hicken et al. (2009)	$0.02^{+0.11}_{-0.02}$	—
Λ CDM	Hicken et al. (2009)	$0.29^{+0.025}_{-0.02}$	—

¹Without calibration

²With calibration

that is not replicated in Λ CDM. The best fit parameters of the FB model are extremely sensitive to small changes in the SNe Ia data as it needs to compensate for these by a large variation in f_{vi} when fitted to an another redshift distribution with a different amount of error on each SNe Ia. In addition, there is a special set of values for f_{vi} which will mimic Λ CDM parameters well, that is the dressed matter density, $\Omega_{m,0}$, in the FB model is a similar value to $\Omega_{m,0}$ derived from Λ CDM.

In the right panel of Figure 3, we have also considered the difference in distance moduli between Λ CDM and FB models (each with the same value of H_0). This is minimised when $0.35 \lesssim \Omega_{m,0} \lesssim 0.4$ for both models. While it is strictly not necessary for the FB model to predict the same value of $\Omega_{m,0}$ as Λ CDM, since its value is not directly observable but inferred from theory, it does so because the SNe Ia sample of Riess et al. (2007) gives a best fit value of $\Omega_{m,0} = 0.34$ for Λ CDM and this happens to fall within this parameter range. Since the best fit parameters for both the Union and Constitution set are $\Omega_{m,0} = 0.29$ and $\Omega_{\Lambda,0} = 0.71$ for Λ CDM, fitting the FB model to this dataset cannot produce a similar value for $\Omega_{m,0}$. In addition, changes in the data that are insignificant for CDM cosmologies will have sufficient leverage to skew the posterior of FB models as a result of the similarity of the solutions when f_{vi} is varied. This behaviour can be attributed to the existence of a tracker solution (Wiltshire 2007b) such that all solutions converge at high redshift, which inhibits the range of possible behaviour that the FB model can exhibit.

5 CONCLUSIONS

One of the more problematic features of the FB model is its failure to provide any predictions on cosmological parameters that are directly observable. Although it produces a value of H_0 that is consistent with the HST Key Project (Freedman et al. 2001) and observations from WMAP when fitted to the Riess et al. (2007) SNe Ia sample with the appropriate Cepheid calibration, this value is not concordant with that obtained from the proper distance

to the sound horizon observed in SDSS (Eisenstein et al. 2005) or the angular scale of the sound horizon observed in WMAP1 (Bennett et al. 2003). Furthermore, the predictions that are made require further investigation of the model at a fundamental level; the constraints from differing SNe Ia sample are discordant. Regardless of the calibration chosen for the Gold sample of Riess et al. (2007), there is no value of H_0 that can produce a consistent cosmology within the FB framework under the currently available SNe Ia data.

The most appealing feature of the FB model is to offer a mechanism for replacing dark energy with an inhomogeneous matter distribution and yet it lacks any formalism by which structure formation can occur. Despite the void fraction being a fundamental parameter of the FB model, it has not been made transparent in either Wiltshire (2007a,b) or Leith et al. (2008) how this is manifested in measurements of clustering statistics or the growth of structure. Indeed, the observational evidence for the FB model presented by Leith et al. (2008) are all derived from geometrical tests and it is not clear what behaviour it would exhibit under more dynamically oriented probes such as large scale structure surveys or constraints from weak lensing. Although it is claimed in Leith et al. (2008) that the ‘concordance’ values of $\Omega_{m,0}$ derived from the FB model are more compatible with those obtained from X-ray measurements of cluster counts (Yepes et al. 2007), it is in fact impossible to state what the FB model would predict without a mechanism for structure formation, since such estimates of $\Omega_{m,0}$ are model dependent. Until the FB model can provide more plausible observational evidence, it is difficult to envisage this model as a serious competitor to Λ CDM when so many questions remain.

ACKNOWLEDGMENTS

The authors wish to acknowledge support from ARC Discover Project DP0665574 and would like to thank Eric Linder for his comments and suggestions.

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